

FINITE SOFT TERMS IN STRING COMPACTIFICATIONS WITH BROKEN SUPERSYMMETRY

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ABSTRACT

We consider the role of supersymmetry breaking soft terms that are present in generalized Narain compactifications of heterotic string theory, in which local supersymmetry is spontaneously broken, with gravitino masses being inversely proportional to the radii of compact dimensions. Such compactifications and their variants, are thought to be the natural application of the Scherk-Schwarz mechanism to string theory. In this paper we show that in the case where this mechanism leads to spontaneous breaking of $N = 4, d = 4$ local supersymmetry, the limit $\kappa \rightarrow 0$ yields a 2-parameter class of distinct, dimension ≤ 3 explicit soft terms whose precise form are shown to preserve the ultraviolet properties of $N = 4$ Super Yang-Mills theory. This result is in broad agreement with that of the field theory Scherk-Schwarz mechanism, as applied to $N = 1, d = 10$ supergravity coupled to Super Yang-Mills, although the detailed structure of the soft terms are different in general.

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Compactifications of superstring theories to four dimensions, in which supersymmetry is spontaneously broken with gravitino masses inversely proportional to the radii of internal tori, have been studied by many authors [1], [2]. Such theories are thought to provide a stringy realization of the well known Scherk-Schwarz mechanism for spontaneously breaking (local) supersymmetry through a generalized dimensional reduction (GDR) procedure [3]. They are particularly interesting in the string context, because there, the number of known supersymmetry breaking mechanisms is relatively small. Although historically it has been difficult to construct realistic low energy models in this way- in particular the difficulty concerning the so called decompactification problem associated with the need for large radii of compactification, there has been recent progress in this area [4], [5]. In any case such a mechanism can provide us with a means of breaking supersymmetry when stringy mechanisms like compactification on orbifolds are not obviously applicable. A particular example of this is the recent application of the GDR mechanism in compactifying M-theory to four dimensions [6]. Interestingly in this case, a connection has been argued to exist between this mechanism and supersymmetry breaking by gluino condensation [7].

In this paper we want explore further the connection between the standard GDR mechanism as applied to field theories, and its stringy counterpart. We will do this by focussing on a rather interesting property of the mechanism in the context of $N = 1, d = 10$ super Yang-Mills (SYM) theory coupled to supergravity compactified to $d = 4$ [8]. In the latter paper it was shown how the explicitly broken $N = 4$ SYM theory obtained in the global limit $\kappa \rightarrow 0$ where gravity decouples, had the remarkable property that the supersymmetry breaking terms were precisely of the form that the authors of [9] had shown preserved the ultraviolet finite properties of the (unbroken) $N = 4$ SYM theory. These finiteness preserving soft terms were described by a 2-parameter set of masses and certain cubic scalar interactions.

Here we want to see if similar results can be obtained in the string case, in particular to the mechanism as applied to Narain type compactifications [10]. Such an analysis will certainly help further understand the connections between field and string theory versions of GDR, because although they share a number of common features, one should be cautious in assuming just how far they are in agreement. Indeed our result will show there is a degree of difference between the two, even though the general conclusion that GDR (in the context of $N = 4$ supersymmetry) produces finiteness preserving soft terms will be verified.

Although one could presumably carry out such an investigation using a number of different formulations of stringy GDR (see refs [1]), we find it attractive to work in the bosonic formalism constructed in [2] which has close connections with the original

Narain construction. We begin then, by introducing a number of relevant formulae appropriate to a discussion of generalized Narain compactifications of the heterotic string with spontaneously broken supersymmetry. The reader is referred to [2] for further details of the construction. The (light cone) world sheet degrees of freedom are: non-compact (spacetime) transverse coordinates $X^a, a = 1, 2$; left/right moving compact coordinates $\bar{X}^i, X^i, i = 1, \dots, 6$, sixteen left moving coordinates $\bar{Y}^I, I = 1, \dots, 16$ together with four right moving complex, compactified “NSR” bosons $H^A, A = 1, 4$ which are equivalent to eight right moving NSR fermions. As usual the spectrum of the theory is obtained from the form of the (right/left moving) Virasoro generators:

$$\begin{aligned} L_0 &= \frac{1}{4}p^2 + L'_0 + N - \frac{1}{2} \\ \bar{L}_0 &= \frac{1}{4}p^2 + \bar{L}'_0 + \bar{N} - 1 \end{aligned} \quad (1)$$

where p is non compact spacetime momentum, N, \bar{N} number operators for right and left moving oscillators of all types. For purely metric compactifications (i.e. the gauge Wilson lines and background antisymmetric tensor fields are set to zero which for simplicity, is the only case we shall consider in this paper) the compact zero mode contributions L'_0 and \bar{L}'_0 take the form

$$L'_0 = \frac{1}{4}p_R^t g^{-1} p_R + \frac{1}{2}\tilde{p}_R^t h^{-1} \tilde{p}_R, \quad \bar{L}'_0 = \frac{1}{4}p_L^t g^{-1} p_L + \frac{1}{2}\tilde{p}_L^t c^{-1} \tilde{p}_L \quad (2)$$

with $p_R = \hat{m} - g\hat{n}$, $p_L = \hat{m} + g\hat{n}$, $\tilde{p}_R = \hat{r} + \hat{t}$, $\tilde{p}_L = \hat{l}$. Here all hatted quantities are integer valued except the shift vector $\hat{t} = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 1)$, which is needed in order that the compact NSR boson correctly describes bosonic and fermionic states. The constant matrix h corresponds to the lattice metric appropriate for the NSR bosons and is given in [2].

The main idea developed in [2] was to show that the above formalism (which at present leads to $d=4$ theories with extended, unbroken supersymmetry), can be generalized to include “Wilson lines” x_i^A that couple the right moving fields H^A to the coordinates X^i in much the same way as ordinary gauge Wilson lines couple the coordinates X^i to Y^I . From now on, unless explicitly stated to the contrary, by Wilson lines we mean those of x_i^A .

The heterotic world sheet action is then

$$\begin{aligned}
I &= \frac{1}{2\pi} \int d\tau d\sigma \{ g_{ij} \partial_{\tilde{a}} X^i \partial^{\tilde{a}} X^j + h_{AB} \partial_{\tilde{a}} H^A \partial^{\tilde{a}} H^B \\
&+ c_{IJ} \partial_{\tilde{a}} X^I \partial^{\tilde{a}} X^J + \epsilon^{\tilde{a}\tilde{b}} x_i^A \partial_{\tilde{a}} X^i \partial_{\tilde{b}} H^A \}
\end{aligned} \tag{3}$$

where $\tilde{a} = 1, 2$ labels the world sheet coordinates, and h_{AB} the metric on the lattice corresponding to the compactified bosons H^A , which is given in [2]. c_{IJ} is the Cartan matrix of $E_8 \times E_8$. Note that in the light cone gauge, the 4 complex right moving NSR worldsheet fermions $\psi^A(z)$, $A = 1, 4$ are given in terms of the chiral bosons $H^A(z)$ by $\psi^A(z) = \frac{1}{\sqrt{2}} e^{iH^A(z)}$. Just as in the case of gauge Wilson lines [10], the presence of the terms in x_i^A in (3) lead to shifts in the zero mode terms in the mode expansions of both X^i and H^A . This is a result of the fact that H^A is subject to the constraint that it be right moving. Standard application of dirac bracket quantization leads to the following x_i^A dependence of the zero modes \dot{q}^A and \dot{q}^i in the expansions of H^A and X^i

$$\dot{q}^A = h^{AB} (p_B - x_{Bi} n^i), \quad \dot{q}^i = g^{ij} (p'_j - x_i^A (p_A - \frac{1}{2} x_{Ak} n^k)) \tag{4}$$

whereas $\dot{q}^I = c^{IJ} p_J$ which appears in the expansion of X^I , is independent of x_i^A . The momenta p'_j, p_B and p_I are all canonical and hence integer valued. The presence of the shift in \dot{q}^A is particularly noteworthy, because as mentioned in [2], it leads to a quantization condition on x_i^A . This follows directly from demanding that the worldsheet supercurrent has NSR boundary conditions and is related to the preservation of Lorentz invariance in the light cone gauge.³

Because of this similarity with gauge Wilson lines, one can, as in the standard case, express the internal zero mode contributions to the 2-d scaling dimension and spin, $H = L'_0 + \bar{L}'_0$, $S = L'_0 - \bar{L}'_0$, in a way which makes manifest the dependence of the latter on the various moduli, including now the parameters x_i^A :

$$H = \frac{1}{2} u^t \chi u, \quad S = -\frac{1}{2} u^t \eta u \tag{5}$$

with

$$\eta = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & c^{-1} & 0 \\ 0 & 0 & 0 & -h^{-1} \end{pmatrix}$$

³This was originally pointed out by I. Antoniadis

$$\chi = \begin{pmatrix} g^{-1} & -g^{-1}b' & 0 & g^{-1}x^t h^{-1} \\ -b^t g^{-1} & (g - b^t)g^{-1}(g - b') & 0 & (g - b^t)g^{-1}x^t h^{-1} \\ 0 & 0 & c^{-1} & 0 \\ h^{-1}xg^{-1} & h^{-1}xg^{-1}(g - b') & 0 & h^{-1} + h^{-1}x \end{pmatrix} \quad (6)$$

where $u^t = (\hat{m}^t, \hat{n}^t, \hat{l}^t, \hat{r}^t + \hat{t}^t)$. In (6), a non vanishing antisymmetric tensor field b has been included via $b' = b - \frac{1}{2}x^t h^{-1}x$, just to make the formal similarity between x_i^A and gauge Wilson lines even more apparent, although from now on we set $b = 0$. The Narain metric η is the standard one except for the addition of the metric.

Following [2] it is convenient to go to a canonical basis where the metrics g, c and h are unit matrices at the expense of defining (moduli) dependent windings and momentum

$$m = e_g^* \hat{m}, \quad n = e_g \hat{n}, \quad \omega = e_h^* \hat{\omega}, \quad l = e_c^* \hat{l} \quad (7)$$

where in (7) $\hat{\omega} = \hat{r} + \hat{t}$ and the matrices e_g, e_c and e_h are chosen so that $g = e_g^t e_g$; $c = e_c^t e_c$; and $h = e_h^t e_h$. Here $*$ represent transpose inverse of a matrix. In this basis (and now setting $b = 0$) H and S take on similar forms as in (5) with vectors \tilde{u} written in terms of the modified windings and momenta (7) and modified matrix $\tilde{\chi}$ given by

$$\tilde{\chi} = \begin{pmatrix} 1 & \frac{1}{2}\mathcal{X}^2 & 0 & \mathcal{X}^t \\ \frac{1}{2}\mathcal{X}^2 & (1 + \frac{1}{2}\mathcal{X}^2) & 0 & (1 + \frac{1}{2}\mathcal{X}^2)\mathcal{X}^t \\ 0 & 0 & 1 & 0 \\ \mathcal{X} & \mathcal{X}(1 + \frac{1}{2}\mathcal{X}^2) & 0 & 1 + \mathcal{X}^2 \end{pmatrix} \quad (8)$$

with $\mathcal{X}_i^A = (e_h^* x e_g^{-1})_i^A$, $\mathcal{X}^2 \equiv \mathcal{X}^t \mathcal{X}$.

In this basis the mass squared and level matching of physical states can be shown to be equivalent to

$$\begin{aligned} \frac{1}{2}M^2 &= \frac{1}{2}(m + \mathcal{X}^t \omega + (1 + \frac{1}{2}\mathcal{X}^2)n)^2 + l^2 + 2\bar{N} - 2 \\ 0 &= -mn - \frac{1}{2}(l^2 - \omega^2) + N - \bar{N} + \frac{1}{2} \end{aligned} \quad (9)$$

With $\mathcal{X}_i^A = 0$, the spectrum (9) describes N=4, d=4 supergravity coupled to super Yang-Mills. Massless spacetime fields within a supermultiplet correspond to states with vectors ω satisfying $\omega^2 = 1$. There are eight physical states with $(\omega^t = \{(\pm 1, 0, 0, 0) + \text{permutations}\})$ describing bosons, whilst $\omega^t = \{(\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}), \text{with even number of minus signs}\}$, describe spacetime fermions. We shall refer to these sets of lattice vectors as ω_B and ω_F respectively. The spectrum (9) generally leads to a spontaneously broken

spectrum when $\mathcal{X}_i^A \neq 0$. Indeed it is easy to see that the four $d = 4$ gravitini ψ_A^μ , $A = 1, 4$ obtain a mass squared $(M_{3/2}^A)^2$ given by

$$(M_{3/2}^A)^2 = \omega^{(A)t} \mathcal{X} \mathcal{X}^t \omega^{(A)}, \quad A = 1, 4, \quad \{\pm \omega^{(A)t}\} = \omega_F \quad (10)$$

where in light cone gauge, the gravitinos correspond to the states

$$\psi_a^A \sim \bar{X}_{-1}^a e^{i\omega^{(A)} \cdot H} |k\rangle \quad (11)$$

k being non-compact, transverse momentum, and it is assumed that the vertex operators in (11), as elsewhere in this paper are normal ordered.

At this point one can see that additional constraints have to be placed on \mathcal{X} in order to avoid giving a mass to the graviton, which should presumably be related to 2-loop modular invariance. Making the basis choice that the lattice vectors $\omega^{(a)t} = (1, 0, 0, 0), (-1, 0, 0, 0), a = 2, 3$ correspond to the 2 transverse directions in $d = 4$, states corresponding to gravitons are of the form $\bar{X}_1^a e^{i\omega^{(b)} \cdot H} |k\rangle$. It is easy to show that the condition of vanishing graviton mass then implies that the first column of \mathcal{X} must vanish, so we take

$$\mathcal{X}_i^{tA} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 \end{pmatrix} \quad (12)$$

where we remind the reader that in the canonical basis, the Wilson line parameters b_1, \dots, d_6 are continuous parameters that are functions of the moduli in general.

In this paper we want to concentrate on the soft terms appearing in the matter sector- i.e. to the adjoint scalars and fermions of N=4 SYM. There are 4 majorana fermions and 6 scalars, which in the present formalism correspond to the states

$$\begin{aligned} e^{il \cdot \bar{Y}} e^{i\omega \cdot H} |k\rangle, \quad \bar{Y}_{-1}^I e^{i\omega \cdot H} |k\rangle, \quad I = 1, \dots, 16; \quad l^2 = 2, \quad \omega \in \Omega_F \\ e^{il \cdot \bar{Y}} e^{i\omega \cdot H} |k\rangle, \quad \bar{Y}_{-1}^I e^{i\omega \cdot H} |k\rangle, \quad I = 1, \dots, 16; \quad l^2 = 2, \quad \omega \in \Omega'_B \end{aligned} \quad (13)$$

where in (13), Ω'_B corresponds to the set of 6 orthogonal vectors Ω_B defined earlier, but with the vectors $\pm(1, 0, 0, 0)$ excluded. It is useful to divide Ω'_B into two parts so that $\Omega'_B = \{\omega^{(\alpha)}, -\omega^{(\alpha)}\}, \alpha = 1, \dots, 3$ in the same way that $\Omega_F = \{\omega^{(A)}, -\omega^{(A)}\}, A = 1, \dots, 4$.

The 6×6 and 4×4 mass matrix squared, M_0^2 and $M_{1/2}^2$ of these states are readily obtained from (9):

$$\begin{aligned} M_0^2 &= \text{Diag}\{\omega^{(\alpha)t} \cdot \mathcal{X} \mathcal{X}^t \cdot \omega^{(\alpha)}, \quad \omega^{(\alpha)t} \cdot \mathcal{X} \mathcal{X}^t \cdot \omega^{(\alpha)}\} \quad \alpha = 1, \dots, 3 \quad ; \\ M_{1/2}^2 &= \text{Diag}\{\omega^{(A)t} \cdot \mathcal{X} \mathcal{X}^t \omega^{(A)}, \quad \omega^{(A)t} \cdot \mathcal{X} \mathcal{X}^t \omega^{(A)}\} \quad A = 1, \dots, 4 \end{aligned} \quad (14)$$

The two fold degeneracy of scalar masses in (14) is indicative that these masses are of ‘ $(A^\alpha)^2 + (B^\alpha)^2$ ’ type, where A^α and B^α are scalar and psuedoscalar fields respectively. Such scalars appear as the internal components of $d = 10$ gauge fields.

An important test of whether explicit supersymmetry breaking masses can preserve certain ultraviolet properties of the unbroken theory, is the vanishing of the graded trace of M^2 , denoted by $\text{GrTr} M^2$, where we adopt the conventions that

$$\text{GrTr} M^2 = \sum_s (-1)^{2s+1} (2s+1) M_s^2 \quad (15)$$

with M_s^2 being the mass squared matrix of the field with spin s .

Since we have set all the gauge Wilson lines to zero, there is no gauge symmetry breaking and so only the adjoint scalars and spinors contribute to $\text{GrTr} M^2$ over the Yang-Mills multiplet. One can now easily check by explicitly substituting the matrix \mathcal{X} into the mass formula (14) that indeed $\text{GrTr} M^2 = 0$ identically. It is interesting to note, by for example relaxing the graviton constraint (12), that the graded trace only vanishes when this condition is implemented. In passing it should be mentioned that the vanishing of $\text{GrTr} M^2$ and its generalizations has been shown to occur within the framework of ‘misaligned supersymmetry’ in string theory, considered in [11]. It would be interesting to see explicitly how the stringy GDR mechanism fits into this.

At this point it is useful to remind the reader some of the results concerning the general structure of soft terms in $N = 4$ SYM that preserve finiteness [9]. Some years ago superspace spurion techniques were employed to address this problem, and the conclusions were as follows. Let $A_\alpha, B_\alpha, \Psi_\alpha, \lambda$, $\alpha = 1, \dots, 3$ and A_μ be the field content of $N=4$ SYM. This choice of field decomposition is natural when writing the theory in terms of $N = 1$ superfields, where the scalars/psuedoscalars A_α, B_α and Majorana fermions Ψ_α lie in $N = 1$ chiral multiplets and the gaugino λ and gauge fields A_μ in $N = 1$ vector multiplets. All fields transform in the adjoint representation of the gauge group G .

The authors of [9] considered the following explicit supersymmetry breaking mass terms:

$$S_{mass}^{(1)} \equiv \frac{1}{C(G)} \int d^4x \text{Tr} \left\{ \frac{1}{2} U_{\alpha\beta} (A_\alpha A_\beta + B_\alpha B_\beta) \right\}$$

$$\begin{aligned}
S_{mass}^{(2)} &\equiv \frac{1}{C(G)} \int d^4x \operatorname{Tr} \left\{ \frac{1}{2} V_{\alpha\beta} (A_\alpha A_\beta - B_\alpha B_\beta) \right\} \\
S_{mass}^{(3)} &\equiv \frac{1}{C(G)} \int d^4x \operatorname{Tr} \{ M_{\alpha\beta} \bar{\Psi}_\alpha \Psi_\beta + \mathcal{M} \bar{\lambda} \lambda \}
\end{aligned} \tag{16}$$

In (16), $V_{\alpha\beta}$ are ‘ $A^2 - B^2$ ’ masses that are known to preserve finiteness [9]. Such masses of course do not contribute to the $\operatorname{GrTr} M^2$. The remaining mass parameters $U_{\alpha\beta}$, $M_{\alpha\beta}$ and \mathcal{M} are subject to the single condition that $\operatorname{GrTr} M^2 = 0$. This condition is a necessary, but not sufficient condition to preserve the finiteness of the explicitly broken $N = 4$ theory. In [9] it was shown that divergences cancel if in addition to the general mass terms discussed above, fixed cubic scalar interaction terms are added, with couplings related to the fermion masses above. That is, one must add

$$\begin{aligned}
S_{cubic}^{(1)} &= \frac{\sqrt{2}}{C(G)} \int d^4x \operatorname{Tr} \{ N_{\alpha\beta\gamma} (A_\alpha (A_\beta A_\gamma - B_\beta B_\gamma) + 2B_\alpha A_\beta B_\gamma) \} \\
S_{cubic}^{(2)} &= \frac{\sqrt{2}}{3!C(G)} \int d^4x \operatorname{Tr} \{ N_0 \epsilon_{\alpha\beta\gamma} (A_\alpha A_\beta A_\gamma - 3A_\alpha B_\beta B_\gamma) \}
\end{aligned} \tag{17}$$

where the parameters $N_{\alpha\beta\gamma}$ and N_0 are given in terms of the fermion masses by

$$N_{\alpha\beta\gamma} = \frac{ig}{2\sqrt{2}} M_{\alpha\delta} \varepsilon_{\delta\alpha\beta}; \quad N_0 = \frac{ig}{2\sqrt{2}} \mathcal{M} \tag{18}$$

and in (17) and (18), Tr indicates the trace over the adjoint representation of the gauge group G , with $C(G)$ the quadratic Casimir, and g the gauge coupling. We choose a normalization of the group generators t^x so that $\operatorname{Tr}(t^x t^y t^z) = (i/2)C(G)f^{xyz}$, $x = 1, \dots, \dim G$, and the coupling g is chosen with the standard normalizations of Yang-Mills terms, (the coupling constant in [9] is $\sqrt{2}$ times ours).

The fact that the soft masses arising from the generalized Narain compactifications satisfy graded sum rules is further evidence that this construction is the natural application of the Scherk-Schwarz mechanism to superstring theory. Having at least shown that one of the conditions for finiteness preserving soft terms discussed above, holds, we now want to consider the role of cubic scalar interactions.

The problem we want to address now is whether such scalar cubics can appear at all in the generalized Narain compactification we are considering here, and secondly to what extent are they the ones required by finiteness of the explicitly broken low energy $N=4$ SYM formally obtained when we take the limit $\kappa \rightarrow 0$. To this end we need to calculate a tree level string scattering amplitude involving three of the ($d = 4$) adjoint scalar fields A_α, B_β . Since these fields are obtained as the 6 internal components of the

10-dimensional gauge fields $A_{\tilde{\mu}}, \tilde{\mu} = 0, \dots, 9$, it is natural to look at the (massless) vector boson emission vertex as a starting point in constructing the (adjoint) scalar emission vertex in the heterotic string. Recall that the vertex describing the emission of an on-shell state with polarization $\Xi^{\tilde{\mu}}(K)$ and momentum $K^{\tilde{\mu}}$, ($K^2 = 0$, and $K \cdot \Xi = 0$), $V_B(\Xi, K)$ is given in the NSR formalism as [12]

$$V_B = \frac{g}{3C(G)} \{ \Xi \cdot \dot{X} - \Xi \cdot \psi K \cdot \psi \} e^{iK \cdot X} \quad (19)$$

where in (19) we have suppressed the left moving vertex operators that describe the gauge quantum numbers of the vector boson. Inclusion of these, and the mass shell condition $K \cdot K = 0$ ensures that V_B is a physical $(1, 1)$ operator. The normalization factor in V_B is needed to reproduce the standard Yang Mills kinetic term $-1/(4g^2 C(G)) \int d^{10}x \text{Tr}(F^2)$. The 6 internal polarizations $\xi^i \equiv \Xi^i$, $i = 4, \dots, 9$, describe adjoint $d = 4$ scalars, if we include the effects of the left moving $E_8 \times E_8$ lattice degrees of freedom by taking ξ^i to transform in the adjoint representation of this group. Decomposing the momentum $K^{\tilde{\mu}} = \{k^{\mu}, p^i\}$ it is easy to see that the 3-scalar scattering amplitude appropriate to standard Narain compactifications is

$$< k_1, p_1, \xi_1 | V_B(k_2, p_2, \xi_2) | k_3, p_3, \xi_3 > = \frac{g}{3C(G)} \text{Tr} \{ \xi_1 \cdot \xi_3 p_3 \cdot \xi_2 + \xi_2 \cdot \xi_1 p_1 \cdot \xi_3 + \xi_3 \cdot \xi_2 p_2 \cdot \xi_1 \} \quad (20)$$

Now it is clear from this that the mass shell condition $k^2 = 0$, $p^2 = 0$ appropriate for toroidal compactification, implies the amplitude (20) vanishes, which is consistent with the fact that unbroken $N = 4, d = 4$ supersymmetry forbids the existence of such cubics. It follows from this that mechanisms that break any of these supersymmetries could generate non vanishing scalar cubic interactions. Thus to compute adjoint scalar emission vertex when $x_i^A \neq 0$ we should take a lead from the gauge boson emission vertex V_B given in (19). In particular the emission vertex V_B has the property that [12]

$$\{G_r, V_0(\Xi, K)\} = V_B(\Xi, K), \quad V_0(\Xi, K) = \Xi \cdot \Psi e^{iK \cdot X}, \quad r = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots \quad (21)$$

where G_r are the oscillators occurring in the expansion of right-moving world sheet supercurrent $J_z(z)$ in the NS sector (once again we suppress the left moving contributions in (21)). In the present formalism generator $J_z(z)$ is given, in the light cone gauge, by

$$J_z(z) = \partial_z \bar{X} e^{iH^1} + \partial_z X e^{-iH^1} + \sum_{\alpha=1}^3 (\partial_z X^{\alpha} e^{-iH^{\alpha+1}} + \partial_z \bar{X}^{\alpha} e^{iH^{\alpha+1}}) \quad (22)$$

with the definitions that $X = \frac{1}{\sqrt{2}}(X^{\mu=2} + iX^{\mu=3})$; $X^{\alpha} = \frac{1}{\sqrt{2}}(X^{i=2\alpha+2} + iX^{i=2\alpha+3})$

The property of the gauge boson emission vertex (21) is essentially a consequence of unitarity. The vertex $V_0(\Xi, K)$ creates conformal dimension $\frac{1}{2}$ states when acting on

the vacuum, when one imposes the mass shell condition. The emission vertex of the ($d = 4$) adjoint scalars that originate from such $d = 10$ gauge fields should be given by a similar construction, except we have to include the effects of having x_i^A non vanishing. In particular, the analogue of $V_0(\Xi, K)$ for these scalars, which we denote $\tilde{V}_0(\xi, \bar{\xi}, k, \bar{k}, x)$ is

$$\tilde{V}_0(\xi, \bar{\xi}, k, \bar{k}, x) = (\xi^\alpha e^{-i\omega^{(\alpha)} \cdot H} + \bar{\xi}^\alpha e^{i\omega^{(\alpha)} \cdot H}) e^{i(\bar{k}^{(\alpha)} X + k^{(\alpha)} \bar{X})} \quad (23)$$

where we have defined complex polarizations (corresponding to wavefunctions of the adjoint scalars) $\xi^\alpha = \frac{1}{\sqrt{2}}(\xi^{i=2\alpha+2} + i\xi^{i=2\alpha+3})$.

One can easily check that the mass shell condition for the 6 adjoint scalars namely $\bar{k}^{(\alpha)} k^{(\alpha)} = -(\mathcal{X}^t \cdot \omega^{(\alpha)})^2$ (c.f.(14)), together with the shifted momentum \dot{q}^i of (4) implies that states created with \tilde{V}_0 have (right- moving) conformal dimension $h = \frac{1}{2}$. (Including the $E_8 \times E_8$ lattice degrees of freedom in \tilde{V}_0 would give $\bar{h} = 1$). The correct dimension (1, 1) emission vertex $V_S(\xi, \bar{\xi}, k, \bar{k}, x)$ is then given by $\{G_r, \tilde{V}_0\}$, where G_r are the modes appearing in the right moving supercurrent with x_i^A non vanishing. In calculating V_S one has to take care about the shifts in the zero modes $\dot{q}^\alpha, \bar{\dot{q}}^\alpha$ appearing in X^α and \bar{X}^α . After some algebra one finds the following expression for the (on-shell) vertex V_S

$$\begin{aligned} V_S(\xi, \bar{\xi}, k, \bar{k}, \mathcal{X}, \bar{\mathcal{X}}) &= \frac{g}{3C(G)} \sum_{\alpha=1}^3 \{ \xi^\alpha \bar{X}^\alpha + \bar{\xi}^\alpha X^\alpha + \sum_{\beta} [\bar{\mathcal{X}}^{t\beta} \cdot \omega^{(\alpha)} (\psi^\beta \bar{\psi}^\alpha \xi^\alpha \\ &\quad - \bar{\psi}^\beta \psi^\alpha \bar{\xi}^\alpha) - \mathcal{X}^{t\beta} \cdot \omega^{(\alpha)} (\bar{\psi}^\beta \bar{\xi}^\alpha - \bar{\psi}^\beta \bar{\psi}^\alpha \xi^\alpha)] \} e^{i\bar{k}^{(\alpha)} X + i k^{(\alpha)} \bar{X}} \\ \text{with } \psi^\alpha &= \frac{1}{\sqrt{2}} e^{i\omega^{(\alpha)} \cdot H} \end{aligned} \quad (24)$$

In (24) we have defined complex Wilson lines $\mathcal{X}_\alpha^A = \frac{1}{\sqrt{2}}(\mathcal{X}_{i=2\alpha+2}^A + i\mathcal{X}_{i=2\alpha+3}^A)$ and we remind the reader that the lattice vectors $\omega^{(\alpha)} = (0, 1, 0, 0)$ etc. As it stands the emission vertex V_S as defined in (24) has a normal ordering problem concerning the bilinear terms in the vertex operators $e^{i\omega^{(\alpha)} \cdot H}$ (or equivalently, the fermions ψ^α). On the face of it there is a similar ordering problem concerning the gauge boson vertex V_B (19). However there, as is well known, the mass shell condition $K \cdot \Xi = 0$ projects out possible pole terms and there is no ordering problem. In the case of V_S however, the mass shell condition by itself is not sufficient and one has to impose further conditions on the Wilson lines. These conditions can be readily obtained

$$\mathcal{X}^{t\alpha} \cdot \omega^{(\alpha)} = 0; \quad \bar{\mathcal{X}}^{t\alpha} \cdot \omega^{(\alpha)} = 0, \quad \alpha = 1..3 \quad (25)$$

i.e the diagonal elements of the (3×3) complex matrix $\mathcal{X}^{t\alpha} \cdot \omega^{(\beta)}$ are required to vanish.

This constraint, together with the one imposed by massless gravitons (12) leads to the following form for the (real) Wilson lines \mathcal{X}_i^A

$$\mathcal{X}_i^{tA} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & 0 & 0 & c_5 & c_6 \\ d_1 & d_2 & d_3 & d_4 & 0 & 0 \end{pmatrix} \quad (26)$$

Armed with V_S it is straightforward to calculate the amplitude

$$\begin{aligned} A_{cubic} &= \langle k_1, \bar{k}_1, \xi_1, \bar{\xi}_1 | V_S(k_2, \bar{k}_2, \xi_2, \bar{\xi}_2, \mathcal{X}, \bar{\mathcal{X}}) | \xi_3, \bar{\xi}_3, k_3, \bar{k}_3 \rangle \\ | \xi, \bar{\xi}, k, \bar{k} \rangle &= \xi^\alpha | \omega^{(\alpha)}, k, \bar{k} \rangle + \bar{\xi}^\alpha | -\omega^{(\alpha)}, k, \bar{k} \rangle \end{aligned} \quad (27)$$

Including the $E_8 \times E_8$ degrees of freedom, one obtains the following explicit supersymmetry breaking trilinear soft terms in the effective action

$$S_{cubic} = \frac{-4gi}{3C(G)} \int d^4x \text{Tr} \sum_{\alpha, \beta=1}^3 \left\{ \frac{3}{2} \mathcal{M}_R^{\beta\alpha} A^\alpha B^\beta B^\alpha + \frac{1}{2} \mathcal{M}_I^{\beta\alpha} A^\alpha A^\beta B^\alpha \right\} \quad (28)$$

where the adjoint (psuedo) scalars (B^α) A^α are defined in terms of the polarizations ξ^α by $\xi^\alpha = B^\alpha + iA^\alpha$. Note that the net result of including the gauge degrees of freedom result is to add an adjoint group index to the polarizations. The mass matrices $\mathcal{M}_R^{\beta\alpha}$ and $\mathcal{M}_I^{\beta\alpha}$ are given by

$$\mathcal{M}_R^{\beta\alpha} = \text{Re}(\mathcal{X}^{t\beta} \cdot \omega^{(\alpha)}) \quad , \quad \mathcal{M}_I^{\beta\alpha} = \text{Im}(\mathcal{X}^{t\beta} \cdot \omega^{(\alpha)}) \quad (29)$$

From the structure of these soft terms, we can see that the $\mathcal{M}_R^{\beta\alpha}$ and $\mathcal{M}_I^{\beta\alpha}$ are parity even (odd) repectively. Having obtained an explicit realization of the soft cubics appearing in generalized Narain compactifications, the final step is to check whether these really are the ones necessary to preserve finiteness of broken $N = 4$ SYM. From our earlier discussion, to do this we need to have an explicit form of the fermion mass matrix. Note that although we know the form of the mass squared matrix (c.f.(14)) the mass matrix requires a bit more work to get in a form that depends linearly on \mathcal{X} . In fact we shall see that there are two (complementary) approaches to deriving this matrix. The first approach is simply to consider the effective $d = 4$ dirac equation for the fermions in the $N = 4$ SYM multiplet, when supersymmetry is broken by \mathcal{X}_i^A . As we have seen (4), the latter parameters generate shifts in the 6 dimensional internal momentum, and one can verify that in this event, the induced 4×4 fermion mass matrix is

$$\begin{aligned}
M_{1/2 AB} &= \sum_{\delta=1}^3 \mathcal{B}_{AB}^{\delta} \operatorname{Re}(\mathcal{X}^{t\delta}) \cdot \omega^{(B)} - \left(\sum_{\delta=1}^3 \mathcal{A}_{AB}^{\delta} \operatorname{Im}(\mathcal{X}^{t\delta}) \cdot \omega^{(B)} \right) \gamma_5 \\
&\equiv M_{1/2 AB}^{(+)} + M_{1/2 AB}^{(-)} \gamma_5
\end{aligned} \tag{30}$$

where the 6 matrices $\mathcal{A}_{AB}^{\delta}$ and $\mathcal{B}_{AB}^{\delta}$ are $SU(2) \times SU(2)$ generators, an explicit representation of which is given in [13]. In obtaining (30) we have used the representation of the $d = 10$ dirac algebra given in [13] and the properties

$$\{\mathcal{A}^{\alpha}, \mathcal{A}^{\beta}\} = \{\mathcal{B}^{\alpha}, \mathcal{B}^{\beta}\} = -2\delta^{\alpha\beta}, \quad [\mathcal{A}^{\alpha}, \mathcal{B}^{\beta}] = 0 \tag{31}$$

Again we see that it is the real and imaginary parts of the complexified Wilson line \mathcal{X}_{α}^A that generates parity even and odd terms -this time via fermion masses.

Now although one might expect that squaring the fermion mass matrix in (30) would give the mass squared formula (14) in fact one does not in general reproduce this. Defining the 6 diagonal matrices, $\Sigma_{\alpha AB} = \operatorname{Re}(\mathcal{X}_{\delta}^t) \cdot \omega^{(B)} \delta_{AB}$ and $\Sigma'_{\beta AB} = -\operatorname{Im}(\mathcal{X}_{\alpha}^t) \cdot \omega^{(B)} \delta_{AB}$, i.e.

$$\begin{aligned}
\Sigma_{1AB} &= \operatorname{Diag}(-c_2 - d_2, c_2 - d_2, -c_2 + d_2, c_2 + d_2) \\
\Sigma_{2AB} &= \operatorname{Diag}(b_4 - d_4, -b_4 - d_4, -b_4 + d_4, b_4 + d_4) \\
\Sigma_{3AB} &= \operatorname{Diag}(b_6 - c_6, -b_6 + c_6, -b_6 - c_6, b_6 + c_6) \\
\Sigma'_{1AB} &= \operatorname{Diag}(-c_1 - d_1, c_1 - d_1, -c_1 + d_1, c_1 + d_1) \\
\Sigma'_{2AB} &= \operatorname{Diag}(b_3 - d_3, -b_3 - d_3, -b_3 + d_3, b_3 + d_3) \\
\Sigma'_{3AB} &= \operatorname{Diag}(b_5 - c_5, -b_5 + c_5, -b_5 - c_5, b_5 + c_5)
\end{aligned} \tag{32}$$

then one finds

$$M_{1/2}^2 = (\mathcal{B} \cdot \Sigma)^2 + (\mathcal{A} \cdot \Sigma')^2 + i\gamma_5 ((\mathcal{B} \cdot \Sigma)(\mathcal{A} \cdot \Sigma') - (\mathcal{A} \cdot \Sigma')(\mathcal{B} \cdot \Sigma)) \tag{33}$$

whereas (14) implies, (after some algebra) that $M_{1/2}^2 = \Sigma \cdot \Sigma + \Sigma' \cdot \Sigma'$ From these definitions, it can be seen that Σ_{α} and Σ'_{α} induce parity even and odd soft terms respectively.

There is another way of viewing the above “mismatch” between (32) and (14). It follows from considering the superconformal algebra in the R sector where one expects that $\{G_0, G_0\} = L_0$. If one focuses on the internal contributions to G_0 and in particular noting the various shifts in momenta induced by \mathcal{X}_i^A , then one again finds an obstruction to the anticommutator of G_0 giving the correct eigenvalues of L_0 , when say we act on the states corresponding to the adjoint fermions considered above. Thus we have to impose further conditions on the Wilson line parameters in order that either the superconformal algebra is satisfied or equivalently that $M_{1/2}^2$ agrees with (14). It is sufficient to impose

$$\{\mathcal{B}^\alpha, \Sigma_\beta\} = 0 \quad , \quad \{\mathcal{A}^\alpha, \Sigma'_\beta\} = 0 \quad , \quad [\mathcal{B} \cdot \Sigma, \mathcal{A} \cdot \Sigma'] = 0 \quad (34)$$

where it is understood that the indices α, β in (34) run over only those values for which the corresponding Σ_α and Σ'_α are non vanishing.

Using the explicit representation of the matrices \mathcal{A}^α and \mathcal{B}^α given in [13], we have found a class of solutions where only one of each matrix Σ_α and Σ'_α as listed in (32) is taken to be non vanishing at a time. This corresponds then to solutions where only 2 of the 12 parameters are turned on at a time. However such solutions are not strictly independent. For example if we take the solution where $\Sigma_3 \neq 0$, all others vanishing, then one can show that the two other solutions where Σ_1 and Σ_2 are turned on separately produce isomorphic theories, in that the fermion mass eigenvalues, scalar masses and trilinear scalar couplings are all equal up to a permutation of the α indices on the fields A_α, B_α respectively. A similar argument applies to the parity odd solutions involving non vanishing Σ'_α , in which there is again only a 2-parameter set of distinct solutions. Therefore the result is that we have a (mutually exclusive) 2-parameter set of parity even and 2-parameter set of parity odd soft terms respectively.

Now we are in a position to finally check if the cubic soft terms appearing in (28) are indeed of the precise form needed for finiteness. Note that the original results by the authors of [9] did not include parity odd soft terms, which we have seen are generated in the generalized Narain compactifications considered here. Thus for the present we set $\Sigma'_\alpha = 0$ and consider parity even terms only.

Consider then the solution to (34) where we take $\Sigma_3 \neq 0$ only. The fermion mass matrix is then

$$M_{1/2} = \frac{1}{2} \begin{pmatrix} 0 & b_6 - c_6 & 0 & 0 \\ b_6 - c_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(b_6 + c_6) \\ 0 & 0 & -(b_6 + c_6) & 0 \end{pmatrix} \quad (35)$$

Diagonalizing gives $M_{1/2}^{diag} = \frac{1}{2}(b_6 - c_6, -b_6 + c_6, b_6 + c_6, -b_6 - c_6)$ where we identify the gaugino mass with $-\frac{1}{2}(b_6 + c_6)$. Note that in this diagonalization procedure, the labels $\alpha = 1..3$ on the scalar and psuedoscalar fields are rotated compared to those before diagonalization. This is easily seen from checking how the yukawa couplings of A_α and B_α change under the orthogonal transformation that diagonalizes $M_{1/2}$. Taking this into consideration, one finds the cubic interactions (28) expressed in terms of the (rotated) fields is

$$S_{cubic} = \frac{4gi}{C(G)} \int d^4x \text{Tr} \{c_6 A_2 B_3 B_1 - b_6 A_1 B_3 B_2\} \quad (36)$$

It can now be checked, using the diagonalized mass matrix $M_{1/2}$ that these agree precisely with those required by the finiteness conditions (17) and (18). As we mentioned, there are also parity violating solutions to (34) which correspond to parity violating soft terms. These have very similar structure to the parity preserving ones, but their effects on the finiteness of $N = 4$ SYM does not seem to have been studied in the literature. It is tempting to speculate that they will turn out to be finiteness preserving as well.

We can make a number of observations about these results. First it is interesting to compare the soft terms we have obtained here in the context of string compactification, and those obtained by applying the standard field theory mechanism to $N = 1, d = 10$ SYM coupled to supergravity [8]. In the latter, a 2 parameter set of (parity even) cubic scalar soft terms and soft masses were also obtained, that preserved the finiteness constraints. Although the form of these terms shows they are roughly similar -they are not identical. In fact there are good reasons why they cannot be in general. The structure of the cubics found in (36) shows that terms trilinear in A_α are not present. From a calculational point of view, this absence was due to the fact that no third rank antisymmetric tensor exists in the formalism, built from \mathcal{X}_α^A , which is needed in order to couple to a term like $\text{Tr} \{A_\alpha A_\beta A_\gamma\}$ which is completely skew in its indices. Contrast this to the field theory situation where such a tensor does exist, namely the structure constants $d_{\alpha\beta\gamma}$ of the ‘flat group’ that is associated with the Scherk-Schwarz mechanism [3], and where indeed trilinear terms in A_α are present. From the viewpoint of finiteness, it is easy to see that such cubics are always proportional to the trace of the fermion mass matrix. But the mass matrix calculated in (30), is always traceless (which follows from the fact that \mathcal{A}^α and \mathcal{B}^β are off diagonal matrices [13]), and so provides further explanation of why such cubics are absent. By contrast, the trace of the fermion mass matrix is not generally zero in the field theory case [8].

In conclusion, we have shown how specific dimension ≤ 3 operators appear in the low energy effective theory obtained from generalized Narain-type compactifications of the heterotic string in which the $N = 4$ local supersymmetry is spontaneously broken. In the global limit $\kappa \rightarrow 0$, one obtains explicitly broken $N = 4$ SYM theory, where the combination of supersymmetry breaking operators were shown to be precisely those which preserve the ultraviolet properties of unbroken $N = 4$ SYM. We have presented the simplest scenario, namely toroidal compactification and no gauge symmetry breaking; a more general study will be presented elsewhere [14]. The realization of the ‘stringy’ GDR mechanism considered here is particularly suitable to generalizations that include orbifolds

of Narain compactifications as stressed in [2]. It would be very interesting to compute the structure of the soft supersymmetry breaking terms in these theories as in the global limit, they will provide examples of explicitly broken $N = 2$ and $N = 1$ theories. (Note that this generalization has no obvious analogue within the usual GDR of $d = 10$ supergravities, which always yields spontaneously broken $N = 4, d = 4$ theories.) In this context, there has been interesting recent work on the partial breaking of $N = 4$ supersymmetry in string theory [15], giving rise to models with $N = 2$ and $N = 1$ supersymmetry. A classification of finiteness preserving soft terms within certain classes of (finite) $N = 2$ supersymmetric field theories has been known for some time [16]. Whether or not the results we have obtained here extend to these theories is worth pursuing.

Finally, GDR mechanisms have been studied recently in the context of M-theory compactified on $M_{10} \times S^1/Z_2$ [6]. These applications are particularly interesting because the theory has bulk and boundary fields living in 11 and 10 dimensions. Since M-theory compactified on a line element is believed to describe strongly coupled $E_8 \times E_8$ heterotic strings [17], one might be able to investigate the effect of large string coupling on our results by computing the soft terms obtained within the framework described in [6].

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